10/24 Recitation

1. Open the **Julia10Determinants** notebook. Complete"Worksheet Problem 1" in the notebook to visualize the 2×2 determinant as area.

2. In class, we proved that the determinant of a diagonal matrix D is the product of the diagonal entries of D. Explain why this makes sense geometrically in the 3×3 case (by viewing the determinant as volume).

3. Show that the 2×2 determinant $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$ satisfies the first two axioms of determinants discussed in class:

   - $\det I = 1$
   - row swaps negate the determinant

4. Find the determinant of $\begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & -3 & -1 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ by reducing to upper triangular form.

5. Open the **Julia10Determinants** notebook.
   (a) Complete "Worksheet Problem 2" in Julia to come up with formulas for the determinant of an $n \times n$ matrix, where $n = 2, 3, 4, 5, 6$. How long does each determinant calculation take?
   (b) Take a look at the 4×4 determinant formula you printed in Julia. Appreciate how tedious it would be to redo problem 3 using this formula. Then really appreciate how systematic it was to use row reduction instead.
   (c) For any n, you can write down a formula for the determinant of an $n \times n$ matrix (as you did for $n = 3, 4, 5, 6$). However, as you probably surmised, these formulas grow exponentially in n. We will see how to work with such formulas tomorrow in lecture. While they are important for theoretical applications, they are not useful in doing concrete calculations. It is much quicker to reduce to upper triangular form as in problem 3 (and this is what programs do).

6. **Coding Challenge**: Do "Worksheet Problem 3" in **Julia10Determinants**.